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From quantum critical to two-channel Kondo physics via charge fluctuations in a quantum dot

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Abstract
We consider charge fluctuations in a quantum dot coupled to an interacting one-dimensional electron liquid. By tuning the coupling between the dot and the one-dimensional electron liquid, one can access the various regimes which arise, which include a quantum critical and a two-channel Kondo regime. The differential capacitance is computed and is shown to contain detailed information about the system.

Nanotechnology has been the source of a renewed interest in the Kondo effect [1]. The incredible progress in miniaturizing solid state devices has made it possible to fabricate small metallic islands (*i.e. quantum dots*) by confining electrons in a two-dimensional electron gas. Quantum dots provide a highly controllable environment in which to study Kondo physics, and allow many aspects of the Kondo effect to be probed.

Most of the studies of the Kondo effect in quantum dots have focused on the case where the dot behaves as a magnetic impurity [1]. However, it has been known for some time that charge fluctuations in a quantum dot can give rise to Kondo physics as well [2, 3]. In this work, we consider charge fluctuations in a quantum dot coupled to an interacting one-dimensional electron liquid. We find the system to have several interesting regimes, including a quantum critical (QC) regime and a two-channel Kondo regime. These regimes can be accessed by tuning the coupling between the dot and the one-dimensional electron liquid; this system could provide a controlled environment in which to probe their properties. In particular, as discussed in detail below, this system provides the remarkable opportunity to directly probe impurity properties of a Kondo system.

It is worth noting this system is not only interesting for its Kondo physics, but also due to its relation/relevance to other systems which have attracted considerable attention [4, 5]. In particular, the fluctuations in the QC regime are similar to the 'local QC fluctuations' which have been argued to describe the behaviour seen in heavy fermion materials [4]. More generally, these fluctuations could be relevant for strongly correlated metals, including doped Mott insulators. This system could provide a controlled environment in which to probe those fluctuations.

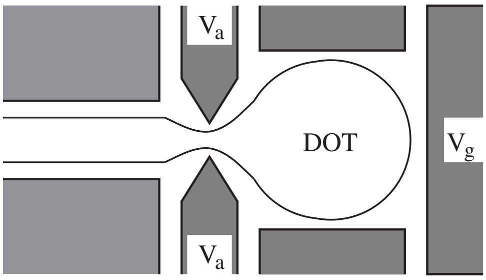

Figure 1. The set-up: a quantum dot coupled to an interacting one-dimensional electron liquid. The number of electrons on the dot is controlled by the gate voltage V_g. A voltage applied to the auxiliary gates, V_a, controls the coupling between the dot and the one-dimensional electron liquid.

The set-up that we consider is shown in figure 1. A large quantum dot is coupled to a reservoir, consisting of an interacting one-dimensional electron liquid. The dot is capacitively coupled to a gate; the gate voltage V_g controls the number of electrons on the dot. The coupling between the dot and the reservoir is controlled by a voltage V_a applied to the auxiliary gates. To model the dot, we assume that the level spacing of the dot is much smaller than any energy scale in the problem; we approximate the spectrum of the dot by a single-particle continuum. Moreover, we consider the case where the reservoir is coupled to the dot via a point contact. The Hamiltonian for the dot has the form $H_{dot} = H_0(\text{dot}) + H_{int}$. $H_0(\text{dot})$ describes the single-particle energy levels of the dot; H_{int} describes the charging energy of the dot, as well as the coupling to the reservoir:

$$H_{int} = \frac{E_c}{2}(\hat{N} - \bar{N})^2 + t(\psi^{\dagger}_{2,s}(0)\psi^{\dagger}_{1,s}(0) + \text{h.c.}). \tag{1}$$

In equation (1), $\psi_{2,s}$ ($\psi_{1,s}$) destroys an electron in the dot (reservoir); $\hat{N}$ is the number operator of the dot; $\bar{N}$ is the optimal number of electrons on the dot, which is proportional to V_g; E_c is the charging energy of the dot; t is the matrix element for tunnelling between the dot and the reservoir, which is controlled by V_a. For generic values of $\bar{N}$, it costs a finite energy to put an extra electron on the dot; for temperatures sufficiently less that E_c, Coulomb blockade develops and the number of electrons on the dot becomes quantized. However, for $\bar{N} = n+1/2$ the energies of the n-electron and $(n + 1)$-electron states are equal, and the charging energy vanishes. Therefore, quantum fluctuations between the dot and the reservoir become important.

In the following, we assume that t is small and we focus on the regime $\bar{N} \approx n + 1/2$. For energies sufficiently less than E_c, the physics will be dominated by the states n and $(n + 1)$. Hence, we can project out all other states and restrict ourselves to this subspace. On considering these states as the two states of a pseudospin $\tau^z = \pm 1/2$ and writing $\hat{N} = (n + 1/2) + \tau^z$, H_{int} takes the form [2]

$$H_{int} = t(\tau^+\sigma^-_{i,j}\psi^{\dagger}_{i,s}(0)\psi^{\dagger}_{j,s}(0) + \text{h.c.}) - h\tau^z, \tag{2}$$

where $h = E_c[\bar{N} - (n+1/2)]$. Equation (2) is a Kondo Hamiltonian with anisotropic couplings. Whereas the Kondo effect usually involves a magnetic impurity, it arises in this system due to charge fluctuations.

Recently, it was argued that, besides the Kondo physics of equation (2), other types of behaviour are possible [6]. In particular, by performing a variational calculation, these authors identified that quantum fluctuations might give rise to tricritical Ising behaviour. However, it remains to be seen whether these results will be confirmed numerically or experimentally.

In this work, we focus on regimes where the system is far from the potential tricritical point, with the result that the Kondo physics dominates.

Being interested in the low energy properties of the system, we expand the electron operator in the reservoir in terms of right and left movers:

$$\psi_{1,\mathrm{s}}(x) = \mathrm{e}^{\mathrm{i}k_\mathrm{F}x}\psi_{\mathrm{R},1,\mathrm{s}}(x) + \mathrm{e}^{-\mathrm{i}k_\mathrm{F}x}\psi_{\mathrm{L},1,\mathrm{s}}(x),$$

where k_F is the Fermi wavevector, and $\psi_{\mathrm{R},1,\mathrm{s}}$ and $\psi_{\mathrm{L},1,\mathrm{s}}$ are the (slowly varying) right and left moving fermion operators. Moreover, upon expanding the electron operator in the dot in harmonics centred about the point contact, the reservoir couples to only a single harmonic [7]. Focusing on that single harmonic, we can write an effective one-dimensional model for the dot [7]. In what follows, we will make extensive use of the boson representation. To do so, the electron operator is written as $\psi_{\mathrm{R/L},i,\mathrm{s}} \sim \mathrm{e}^{\pm\mathrm{i}\sqrt{4\pi}\phi_{\mathrm{R/L},i,\mathrm{s}}}$ where the chiral fields, $\phi_{\mathrm{R},i,\mathrm{s}}$ and $\phi_{\mathrm{L},i,\mathrm{s}}$, are related to the usual Bose field $\phi_{i,\mathrm{s}}$ and its dual field $\theta_{i,\mathrm{s}}$ by $\phi_{i,\mathrm{s}} = \phi_{\mathrm{R},i,\mathrm{s}} + \phi_{\mathrm{L},i,\mathrm{s}}$ and $\theta_{i,\mathrm{s}} = \phi_{\mathrm{R},i,\mathrm{s}} - \phi_{\mathrm{L},i,\mathrm{s}}$. It will also prove useful to form *charge* and *spin* fields $\phi_{i,\rho/\sigma} = (\phi_{i,\uparrow} \pm \phi_{i,\downarrow})/\sqrt{2}$. In terms of these variables,

$$\begin{aligned} H(\mathrm{lead}) + H_0(\mathrm{dot}) &= \frac{v_\mathrm{F}}{2}\sum_{i=1}^{2}\int_{-\infty}^{0}\mathrm{d}x\,(\partial_x\theta_{i,\sigma})^2 + (\partial_x\phi_{i,\sigma})^2 \\ &+ \frac{v_\mathrm{F}}{2}\sum_{i=1}^{2}\int_{-\infty}^{0}\mathrm{d}x\,K_i(\partial_x\theta_{i,\rho})^2 + \frac{1}{K_i}(\partial_x\phi_{i,\rho})^2. \end{aligned} \tag{3}$$

The Luttinger parameter in the reservoir, K_1, is determined by the interactions—$K_1 < 1$ for repulsive interactions and $K_1 > 1$ for attractive interactions. For the dot, $K_2 = 1$. In this work, we will focus on the case of repulsive interactions, $K_1 < 1$. To analyse the physics it will prove useful to unfold the system, and work solely in terms of right moving fields [8]. Moreover, by forming linear combinations of the Bose fields in the dot and the reservoir, the system can be treated as two identical Luttinger liquids with an effective Luttinger parameter [7]

$$K = \frac{2K_1}{K_1+1}. \tag{4}$$

The effects of equation (2) can be deduced by a renormalization group (RG) analysis. More generally, we will consider

$$H_\mathrm{int} = t(\tau^+\sigma^-_{i,j}\psi^\dagger_{i,\mathrm{s}}(0)\psi^\dagger_{j,\mathrm{s}}(0) + \mathrm{h.c.}) + t'\tau^z\sigma^z_{i,j}\psi^\dagger_{i,\mathrm{s}}(0)\psi^\dagger_{j,\mathrm{s}}(0) - h\tau^z. \tag{5}$$

Though the t' term is not present in equation (2), it will be generated upon renormalization. To lowest non-trivial order, the RG equations for the parameters are

$$\frac{\mathrm{d}\lambda^{xy}}{\mathrm{d}l} = \frac{1}{2}\left(1-\frac{1}{K}\right)\lambda^{xy} + \frac{1}{K}\lambda^{xy}\lambda^z, \qquad \frac{\mathrm{d}\lambda^z}{\mathrm{d}l} = (\lambda^{xy})^2, \qquad \frac{\mathrm{d}\lambda^h}{\mathrm{d}l} = \lambda^h - K(\lambda^{xy})^2\lambda^h, \tag{6}$$

where $\lambda^{xy} \sim t$, $\lambda^z \sim t'$, and $\lambda^h \sim h$. The RG flows in the λ^z–λ^{xy} plane are plotted in figure 2. Notice that there is a *QC point* occurring when $\lambda^{xy}_\mathrm{c} \equiv (1-K)/(2\sqrt{K})$. For $\lambda^{xy} \leqslant \lambda^{xy}_\mathrm{c}$ the coupling flows to zero, while for $\lambda^{xy} > \lambda^{xy}_\mathrm{c}$ the system flows to strong coupling. For $\lambda^{xy} \leqslant \lambda^{xy}_\mathrm{c}$, the system flows to the fixed point where the dot is decoupled from the reservoir. (We will refer to this as the *decoupled fixed point*.) In terms of the effective Kondo model, the 'impurity' is unscreened at low energies. For $\lambda^{xy} > \lambda^{xy}_\mathrm{c}$, λ^{xy} initially decreases under the RG. However, it will eventually start to increase and then flow off to strong coupling. Integrating equation (6), we find that $\lambda^{xy} = \mathcal{O}(1)$ at a scale

$$T_\mathrm{K} = E_\mathrm{c}\exp\left[\frac{-1}{|\delta|}[\arccos(|\delta|) - \arctan(x_0/|\delta|)]\right], \tag{7}$$

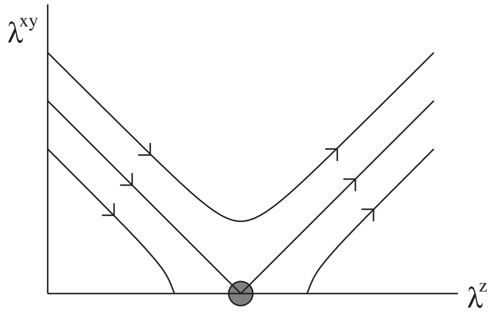

Figure 2. RG flows of equation (6).

where $x_0 = (K-1)/(2K)$ and $|\delta| = \sqrt{\{(\lambda^{xy})^2/K - x_0^2\}}$. For energies below T_K, the dot and the reservoir are strongly coupled. The strong coupling fixed point which arises is non-trivial—it corresponds to the *two-channel Kondo fixed point* with a spin-1/2 impurity [3]. This occurs because both spin-up and spin-down electrons in the reservoir try to occupy the single available charge state on the dot.

It should be noted that a related system was considered recently in [9]. In that work, the authors considered a resonant level coupled to a Luttinger liquid of spinless fermions. If the Luttinger parameter was smaller than some critical value, $K < K_c$, they too found a transition as one tuned the coupling between the dot and the Luttinger liquid. The authors of [9] focused on the zero-temperature properties of their system. In this work, we show that much rich physics can be observed at finite temperatures and frequencies.

The quantity of experimental interest is the differential capacitance. In terms of the effective Kondo model, this corresponds to the impurity susceptibility [2, 3]. Hence, we will need to calculate correlation functions of impurity operators. To begin with, we will focus on the regime where $\lambda^{xy} < \mathcal{O}(1)$. In this regime, we can calculate the impurity susceptibility using the RG. In general, an N-point impurity correlation function $G_N(\tau_1, \ldots, \tau_N; \lambda_i, E_c) \equiv \langle \tau^z(\tau_1) \ldots, \tau^z(\tau_N) \rangle$ satisfies the RG equation

$$\left[\frac{\partial}{\partial l} + \sum_i \beta_i \frac{\partial}{\partial \lambda_i} + N\gamma\right] G_N(\tau_1, \ldots, \tau_N; \lambda_i, E_c) = 0, \tag{8}$$

where $\beta_i = \mathrm{d}\lambda_i/\mathrm{d}l$, and γ is the anomalous exponent. The solution of equation (8) is

$$G_N(\tau_1, \ldots, \tau_N; \lambda_i, E_c) = \exp\left[-N \int_0^{l^*} \mathrm{d}l\, \gamma(l)\right] G_N(\tau_1, \ldots, \tau_N; \lambda_i(l^*), \mathrm{e}^{-l^*} E_c).$$

Using equation (6), we obtain

$$G_N(\tau_1, \ldots, \tau_N; \lambda_i, E_c) = \mathrm{e}^{-NK(1-K)/2} \mathrm{e}^{-NK^2 f(l^*)} G_N(\tau_1, \ldots, \tau_N; \lambda_i(l^*), \mathrm{e}^{-l^*} E_c), \tag{9}$$

where

$$f(l^*) = |\delta| \left[\frac{y_0^2 + (|\delta| - x_0)^2 \mathrm{e}^{2|\delta| l^*}}{y_0^2 - (|\delta| - x_0)^2 \mathrm{e}^{2|\delta| l^*}}\right] \qquad (\lambda^{xy} < \lambda_c^{xy}), \tag{10a}$$

$$f(l^*) = \frac{x_0}{1 - x_0 l^*} \qquad (\lambda^{xy} = \lambda_c^{xy}), \tag{10b}$$

$$f(l^*) = |\delta| \tan\left[|\delta| l^* + \arctan\left(\frac{x_0}{|\delta|}\right)\right] \qquad (\lambda^{xy} > \lambda_c^{xy}). \tag{10c}$$

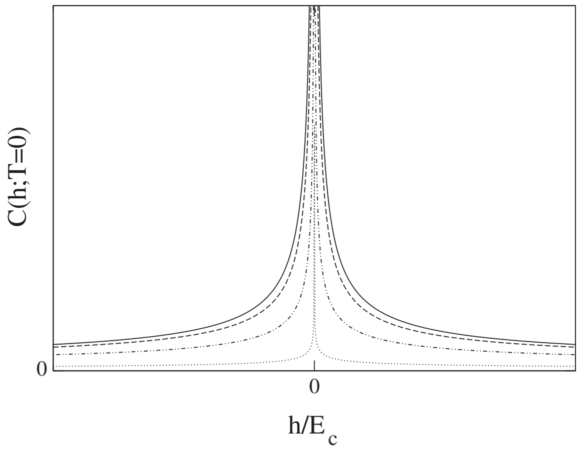

Figure 3. $C(h; T = 0)$ versus h for $\lambda^{xy} \leqslant \lambda_c^{xy}$ (with $K = 0.6$). Solid curve: $|\delta| = 0$; dashed curve: $|\delta| = 0.1$; dash–dotted curve: $|\delta| = 0.2$; dotted curve: $|\delta| = 0.3$.

In equation (10), $x_0 = (K-1)/(2K)$; $y_0 = \lambda^{xy}/\sqrt{K}$ and $|\delta| = \sqrt{\{x_0^2 - y_0^2\}}$ in equation (10a); $|\delta| = \sqrt{\{(\lambda^{xy})^2/K - x_0^2\}}$ in equation (10c). Choosing $e^{l^*} \sim E_c/E$, the correlation function on the right-hand side of equation (9) can be evaluated perturbatively.

We start by considering the temperature dependence of the differential capacitance on resonance, $C(h \to 0; T)$. Using equation (9) with $N = 2$, we obtain

$$C(h \to 0; T) = \frac{e^{K(K-1)}}{4T} e^{-2K^2 f(T)}, \tag{11}$$

where $f(T)$ is given by equation (10) with $l^* = \ln(E_c/c_1 T)$ (c_1 is an $\mathcal{O}(1)$ constant). From equation (11), we see that $C(h \to 0; T) \sim T^{-1}$ near the decoupled fixed point, $T \to 0$ for $\lambda^{xy} \leqslant \lambda_c^{xy}$. However, it is worth noting there is an additive logarithmic correction when $\lambda^{xy} = \lambda_c^{xy}$: $C(h \to 0; T) \sim T^{-1}[1 + 2K^2/\ln(E_c/c_1 T)]$. Moreover, in the QC regime ($\lambda^{xy} = \lambda_c^{xy}$ for $T \gg E_c \exp[2K/(K-1)]$), $C(h \to 0; T) \sim T^{\Delta_T - 1}$ where $\Delta_T = (1-K)^2/2$.

It is also interesting to consider the differential capacitance at $T = 0$ as a function of gate voltage, $C(h; T = 0) = E_c\, d\langle \tau^z \rangle / dh$. Using equation (9) with $N = 1$, we obtain

$$\langle \tau^z \rangle = \pm \frac{e^{K(K-1)/2}}{2} e^{-K^2 f(|h|)}, \tag{12}$$

where the plus (minus) sign is for $h > 0$ ($h < 0$), and $f(|h|)$ is given by equation (10) with $l^* = \ln(E_c/c_2|h|)$ (c_2 is another $\mathcal{O}(1)$ constant). The differential capacitance versus gate voltage is plotted in figure 3. Near the decoupled fixed point, we find $C(h; T = 0) \sim |h|^{2|\delta|-1}$ as $|h| \to 0$ ($|\delta|$ is as in equation (10a)). When $\lambda^{xy} = \lambda_c^{xy}$, similarly to $C(h \to 0; T)$, $C(h; T = 0)$ receives logarithmic corrections. However, this time the logarithmic correction is multiplicative: $C(h; T = 0) \sim 1/[|h| \ln^2(E_c/c_2|h|)]$. Also, in the QC regime ($|h| \gg E_c \exp[2K/(K-1)]$), $C(h; T = 0) \sim |h|^{\Delta_h - 1}$ where $\Delta_h = (1-K)^2/4$.

Notice that near the decoupled fixed point, $C(h \to 0; T) \sim T^{-1}$ as $T \to 0$. This Curie–Weiss-like form arises because the 'impurity' behaves basically like a free spin. However, the local 'moment' is reduced from its non-interacting value. From equation (12), we see that the amount by which the local 'moment' is reduced depends on K, as well as the value of λ^{xy}. Also, in the QC regime, the differential capacitance has power-law behaviour; the exponents satisfy $\Delta_T = 2\Delta_h$.

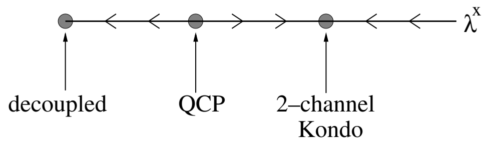

Figure 4. Fixed points and RG flows.

Now, we consider the physics in the regime $\lambda^{xy} > \lambda_c^{xy}$ for energies below T_K (with T_K given by equation (7)). In this regime, the system is close to the two-channel Kondo fixed point. To proceed, we follow [10] and form combinations of the fields in the dot and the reservoir: $\phi_{R,c}$, $\phi_{R,sp}$, $\phi_{R,f}$, and $\phi_{R,sf}$. Then, we perform the unitary transformation, $U = \exp(i\sqrt{4\pi/K}\tau^z\phi_{R,f}(0))$, which ties charge to the 'impurity'. Finally, we introduce new fermion fields, $d \sim \tau^-$, $X \sim e^{i\sqrt{4\pi}\phi_{R,sf}}$, and $f \sim e^{i\sqrt{4\pi}\phi_{R,f}}$. Upon performing these transformations, H_{int} becomes

$$\begin{aligned} H_{int} &= v_F\tilde{\lambda}^{xy}(d^\dagger + d)(X^\dagger(0) - X(0)) \\ &\quad + v_F\sqrt{4\pi/K}(\tilde{\lambda}^z - 1)(d^\dagger d - 1/2)f^\dagger(0)f(0) - v_F\tilde{\lambda}^h(d^\dagger d - 1/2), \end{aligned} \tag{13}$$

where $\tilde{\lambda}^{xy}$, $\tilde{\lambda}^z$, and $\tilde{\lambda}^h$ are the renormalized values of the couplings.

Using equation (13), we can calculate the differential capacitance near the two-channel Kondo fixed point. Starting with the differential capacitance on resonance, we find (ignoring the irrelevant $(\tilde{\lambda}^z - 1)$ term)

$$C(h \to 0; \omega, T) = \frac{1}{T_K}\int \frac{dx}{2\pi}\tanh\left(\frac{xT_K}{2T}\right)\frac{1}{x^2+1}\frac{1}{x - (\omega/T_K) - i0^+}. \tag{14}$$

For $\omega = 0$ and $T \ll T_K$, this reduces to the well-known result for the impurity susceptibility of the two-channel Kondo model $C(h \to 0; T) = 1/(\pi T_K)\ln(T_K/T)$. We can also calculate $C(h; T = 0)$. Using equation (13) (ignoring the irrelevant $(\tilde{\lambda}^z - 1)$ term),

$$\langle\tau^z\rangle = \frac{h}{T_K}\int \frac{dx}{2\pi}\tanh\left(\frac{xT_K}{2T}\right)\frac{x}{(x^2 - (h/T_K)^2)^2 + x^2}. \tag{15}$$

For $T = 0$ and $|h| \ll T_K$, $C(h; T = 0) = 4/(\pi T_K)\ln(T_K/|h|)$. Notice that $C(h \to 0; T)$ ($C(h; T = 0)$) diverges as $T \to 0$ ($|h| \to 0$). However, the divergence in this case is weaker than what occurs near the decoupled fixed point. This is because, near the two-channel Kondo fixed point, charge is tied to the 'impurity'. As a result, the ground states $\tau^z = 1/2$ and $-1/2$ are orthogonal, in that they are not connected by τ^+ or τ^- [11]. This removes the power-law divergence which occurs near the decoupled fixed point, and replaces it with the weaker logarithmic divergence.

In the above discussion, we saw three fixed points arise (shown schematically in figure 4): (1) the decoupled fixed point, (2) a QC point, and (3) the two-channel Kondo fixed point. The QC point and the two-channel Kondo fixed point are particularly interesting because they are non-trivial scale invariant fixed points. As a consequence, one should be able to observe ω/T scaling near these fixed points by applying an AC component to the gate voltage. More specifically, we expect the dynamical capacitance on resonance to have the form

$$C(h \to 0; \omega, T) = T^{\nu-1}X(\omega/T). \tag{16}$$

In the QC regime, we can calculate the scaling function $X(\omega/T)$ for $(1 - K) \ll 1$. In the leading logarithm approximation, we find $\nu = \Delta_T$ ($\Delta_T = (1 - K)^2/2$) and

$$X(\omega/T) = \frac{1}{4\pi}\left(\frac{2\pi}{E_c}\right)^\nu \sin\left(\frac{\pi\nu}{2}\right)B\left(\frac{\nu}{2} - i\frac{\omega}{2\pi T}, 1 - \nu\right)$$

where B is the beta function [12]. Near the two-channel Kondo fixed point, we use equation (14) to obtain $\nu = 1$ and

$$X(\omega/T) = \frac{1}{\pi T_{\mathrm{K}}} \ln\left(\frac{T_{\mathrm{K}}}{\max(\omega, T)}\right) + \frac{\mathrm{i}}{2T_{\mathrm{K}}} \tanh\left(\frac{\omega}{2T}\right).$$

Note that $X(\omega/T)$ is, in general, complex. Therefore, the differential capacitance will have components both in phase and out of phase with the gate voltage.

To summarize, we considered charge fluctuations in a (large) quantum dot coupled to an interacting one-dimensional electron liquid. By tuning the coupling between the dot and the one-dimensional electron liquid, one can access the various regimes that arise, which include a QC and a two-channel Kondo regime. Moreover, this system provides the remarkable opportunity to directly probe impurity properties of a Kondo system via differential capacitance measurements. As the differential capacitance of a large quantum dot has recently been measured [13], we are hopeful that the physics described in this work will be observed in the near future.

Acknowledgments

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